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Albers, Scott

University of Missouri School of Law

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# Of Jane Austen and the Secret Life of Econometric Quantities

*or, as otherwise entitled:*

## On Okun's Law and "the Multiplicative Inverse Surprise"

By Scott A. Albers\*

For Emma Thompson

**Abstract:** *This article proposes that Okun's Law is an empirical relationship between employment and production which, in the United States, correlates to the relationship between the radius of a circle and one-half of its circumference i.e. numerically, the ratio  $1 : \pi$ . This requires two new sets of numbers, these being the set of feminine numbers ( $0 < F < 1$ ) and their inverses in the set of masculine numbers ( $1 < M$ ), as well as the invention of the "Jane Austen Multiplicative Inverse." Data describing Okun's Law appears to confirm the reliability of this approach with an accuracy of up to 1.05 parts in 100,000.<sup>1,2</sup>*

### Introduction

But Elinor — how are HER feelings to be described? — From the moment of learning that Lucy was married to another, that Edward was free, to the moment of his justifying the hopes which had so instantly followed, she was every thing by turns but tranquil.

Jane Austen, *Sense and Sensibility*

Emma Thompson is a well-known actress of stage and screen, winning the 1995 Academy Award for Best Adapted Screenplay and the 1995 Golden Globe Award for Best Supporting Actress in her performance as Miss Elinor Dashwood, the stalwart and sensitive heroine of Jane Austen's 1811 novel *Sense and Sensibility*. One of the most memorable scenes of this film was Ms. Thompson's engaging portrayal of Miss Dashwood's relief unto tears on learning that Mr. Edward Ferrars was *not* married.

From this simple association arose the idea of a "Jane Austen Multiplicative Inverse," a pedagogic device presented here as nothing more than hopefully helpful. Miss Thompson's admirable portrayal of intelligent, thoughtful and independent women in the films *Henry V*, *Much Ado About Nothing*, *Last Chance Charlie* and *Sense and Sensibility* have led me to hope

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\* Scott Albers is a criminal defense attorney practicing law in Northwest Montana, U.S.A. (1994 –present), and previously Missouri (1986 - 1994). He obtained a Juris Doctorate degree in 1986 at the University of Missouri School of Law in Columbia and maintains a long-term interest in international law and macroeconomics. He may be contacted at [scott\\_albers@msn.com](mailto:scott_albers@msn.com).

<sup>1</sup> This article contains 4,612 words, with an abstract of 99 words. Additional papers toward an understanding of the ideas presented herein is found at "scribd\_scott\_albers\_1" and the Munich Personal Repository.

<sup>2</sup> Acknowledgements. This reply to Dr. Edward Knotek's rhetorical question "How Useful is Okun's Law?" (Economic Review 2007) is possible only because he has been so generous with his time, his valuable information, his insights as to Okun's Law and his repeated explanations of data and methods which he used in that article.

that she will not object to her name being associated with a mathematics using a feminine analogy, not unlike the “male” and “female” aspects of electrical circuits. This essay is composed stylistically along the lines of Ms. Thompson’s speech at the Golden Globes ceremony which perhaps the reader may find, if not as talented her own, at least worthy of an American.

With this introduction I proceed by requesting that the reader concede a single, fundamental point, and it is this: *All macroeconomic quantification relies first upon the theory of numbers.* I maintain that this point is at the very essence of mathematical economics.

### **The Realm**

Should one tour the terrain of number theory for any even brief period of time one must admit that the denizens of that kingdom are decidedly Victorian in disposition. Nowhere is this tendency more evident than in the theory of the multiplicative inverse.

Where else in all the civilized world of thought does the subjected and meek “Miss  $\frac{1}{2}$ ” marry forever the redoubtable and dependable “Mr. 2” and emerge through all time and eternity as the complete inverse couplet: “ $\frac{1}{2} \times 2 = 1$ ”? In a fashion worthy of Jane Austen herself the feminine numbers ( $0 < F < 1$ ) find their multiplicative inverses in the masculine numbers ( $1 < M$ ) and through simple association verging on osmosis render the theoretic, nay required, association  $\frac{1}{x} \times x \times \frac{x}{1} = 1$ . A proposition of this sort found in Nature Herself is hard to imagine, and yet we all feel that such is, or at least should be, so the case.

But what of the eccentric and even erratic “Mr. 6.28...” and the fluttering heart of “Miss  $\frac{1}{2}$ ” when he draws nigh? Does it not occur to even the most pedantic among us that, through a once-upon-a-time and probably – even ‘likely’ – illicit conjoining of the two emerged the creation of “Mr.  $\pi$ ” himself in all his wild, dissolute and raven-haired *gloire*? Here let us begin our tale of the secret heart and life of numbers and the macro-econometric quantities which they convey, their quiet joys and their surprises.

### **The Women of the Realm**

First be it mentioned in what must inevitably pose a strange and dark quest that an inverse may be created for any feminine ( $0 < F < 1$ ) number by the simple expedient of reversing in numerator and denominator her own character. In this sense every  $\frac{1}{x} = F$  must and always will have some number  $x/1 = M$  to whom she can cling, attach and render the complete couplet:

$$\frac{1}{x} \times x \times \frac{x}{1} = 1$$

... to which her fancies and thoughts so inevitably tend.

Here let the word “Progenic” be introduced, as referring to the product of the above association of feminine and masculine numbers. By “progenic” (“P” as taken from the root word “progeny” signifying “child” or “children”) I mean the number which is derived from two other numbers *as an intended result*, as contrasted with a number which appears in the data through statistical chance.

For beyond this simple number “1” are the distant imaginings of other things, all of which are themselves also “progenic” yet as far unlike “1” as sea is to shore. In this fashion, using “1” as the fulcrum of her desire, every feminine number may create a “Jane Austen multiplicative inverse” by simply permitting herself the chance. While such combinations may be dismally sanctioned by the best society, they have not as yet been scandalized successfully out of existence.

For example, should a “Jane Austen multiplicative inverse” be derived for the number  $\frac{1}{2}$  about the progenic number  $\pi$ , she need only to calculate the likely quantity of  $\frac{2}{1} \times \pi = 2\pi$ , and through her own multiplicative effort as conjoining to this number, render the subsequent progenic  $\pi$  as follows:

$$\begin{array}{lcl} \frac{1}{2} & \longrightarrow & \pi \\ & & \pi \times \frac{2}{1} = 2\pi \\ \frac{1}{2} \times 2\pi & = & \pi \end{array}$$

And so I lay it down in bold: a *proper* multiplicative inverse has as its progenic product the number “1,” and a *Jane Austen* multiplicative inverse has as its progenic product some number greater than 1, some “P,” implying thereby the existence of some masculine father as determined to be always at some multiple greater than  $x/1$ .

The *gravitas* of the question is obvious.

Let us consider the simple process whereby a Jane Austen multiplicative inverse may be procured for the number  $\frac{1}{46}$  about the progenic number  $\phi = 1.6180\dots$  We would use the following straight-forward calculus:

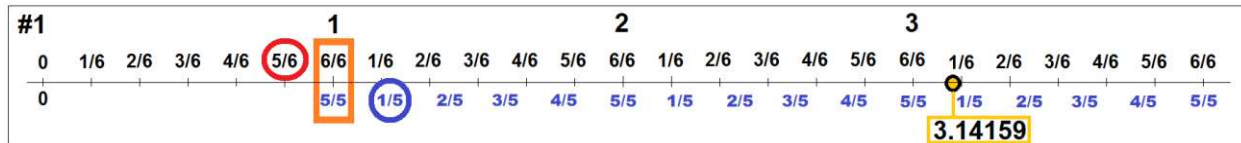
$$\begin{array}{lcl} \frac{1}{46} & \longrightarrow & \phi \\ & & \phi \times \frac{46}{1} = 46\phi \\ \frac{1}{46} \times 46\phi & = & \phi \end{array}$$

This of course does not diminish in the least the culpability of the affair, nor should it properly challenge the loyalty of  $\frac{1}{46} \times \frac{46}{1} = 1$ . It merely serves to show that a world of longing exists in even the purest heart of number theory, one as irrevocable as The Dawn.

There are of course corollaries to this situation and not all feminine numbers are of the straight-forward  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ , etc. sort. Nevertheless a more intractable feminine number, say “Miss  $\frac{5}{6}$ ,” has but to reverse the numerator and denominator of her own character and straight-away “Mr.  $\frac{6}{5}$ ” shall appear, to render the equation complete (see #1, below):

$$\frac{5}{6} \times \frac{6}{5} = 1$$

As this might be placed on a number line, we have:



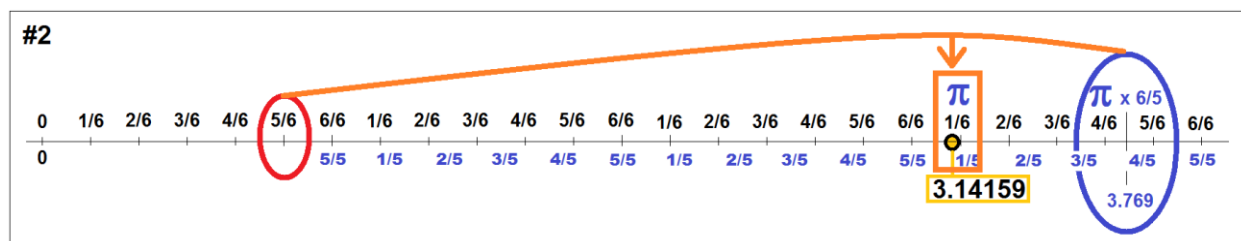
Because it will be quite important at a later point in this story, we might consider the manner in which “Miss 5/6” might aspire to the progenic number “Mr.  $\pi$ ” as a matter of some strategy. Multiplying  $\pi \times 6/5$  yields the following (see #2, below):

$$\frac{5}{6} \longrightarrow \pi$$

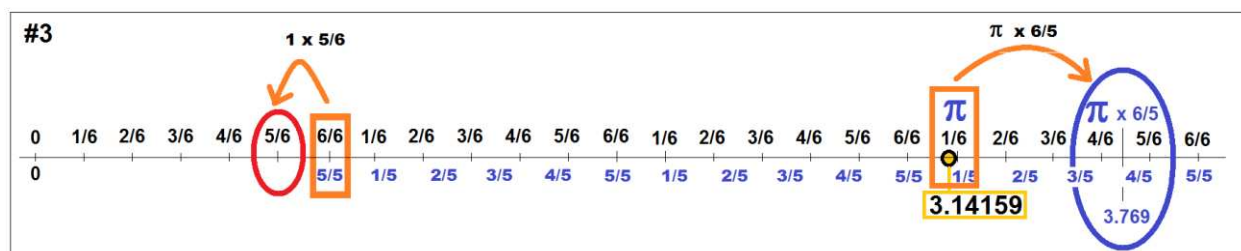
$$\pi \times \frac{6}{5} = \frac{6}{5} \pi$$

$$\frac{5}{6} \times \frac{6}{5} \pi = \pi$$

Or stated in the context of a number line:



In this day and age of relativistic considerations one might inquire as to how this all looks from the point of view of the egocentric and generally clueless progeny of “Miss 5/6” and her secret lover, the mysterious “Mr.  $6\pi/5$ ”. Because the progenic number “Mr.  $\pi$ ” is as yet without mate or prospect, only the numbers 1 and  $\pi$  exist for him. As depicted below, from his point of view his mother, the feminine “Miss 5/6,” simply counts for “ $5/6 \times 1$ ,” representing his own egotism multiplied by what he sees in her. Conversely his father represents “ $6/5 \times$  himself.” (see #3, below):



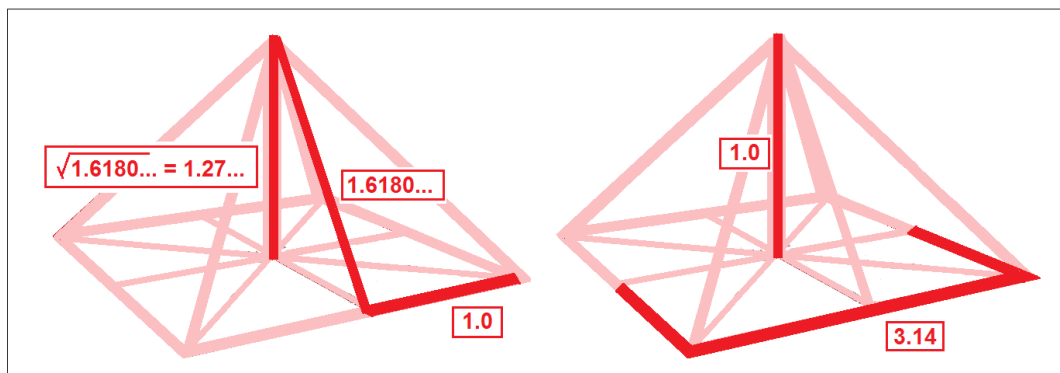
The fact that this relationship might be expressed in decimals rather than fractions does not alter the situation in the least, however modern it might appear at first. Nevertheless the translation of one set of ideas into a basically foreign set of ideas is not without its problems. In any event the following example, using decimals, is equivalent for the purposes of this commentary on the heart of logic, to wit:

$$0.8333... \times 1.2 = 1$$

But, says the macro-economist, what has this to do with me, or with the proper function of my trade in calculating macro-economic quantities of any sort? To this answer turn we now.

### The Egyptian Affair

Once in Egypt, not far from the ancient and exotic city of Cairo, known to the Arabs as “the Vanquisher” and “the city of a thousand minarets,” an enormous sculpture was raised upon the Giza Plateau, the Great Pyramid and its set of consorting pyramids, the word itself meaning in Greek “fire in the center.” And through all the years hence the mathematical properties of this figure have guided scientists, mystics and the common spectator tourist alike to a healthy admiration of the capacities of others not themselves. Among the properties of the greatest of these pyramids we find both  $\pi$  ( $= 3.14159...$ ) and  $\phi$  ( $= 1.61803...$ ) as the two closest of companions, as follows:



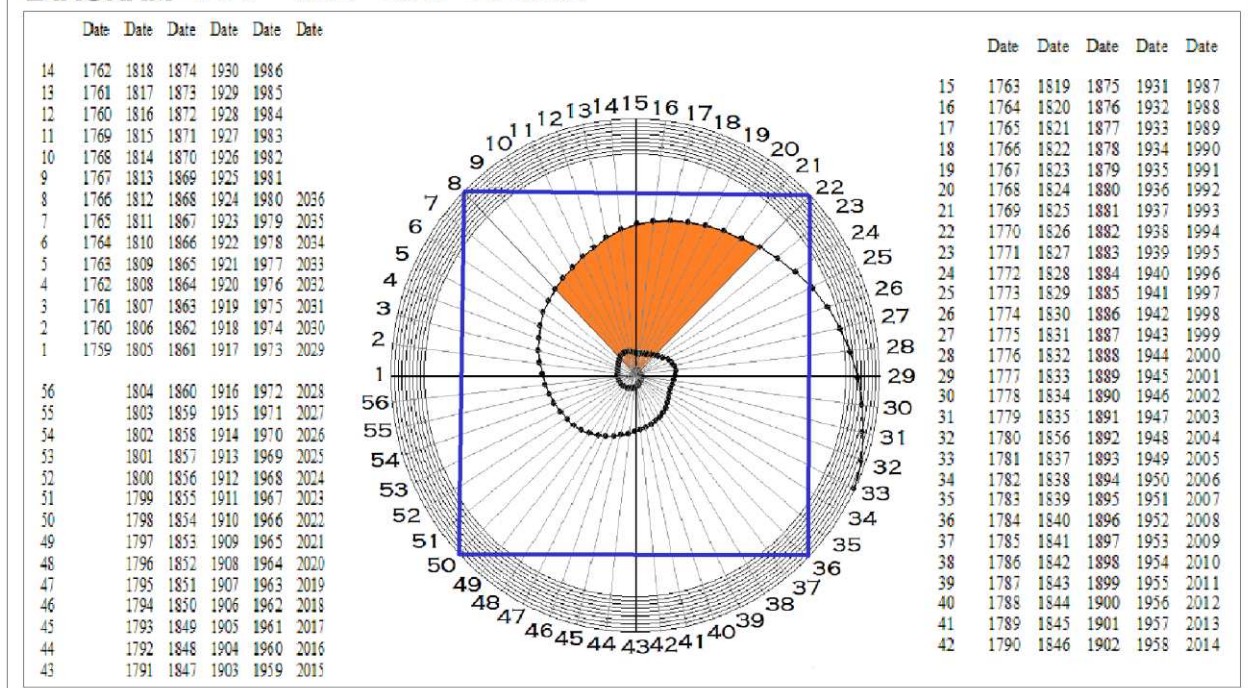
The significance of this monument of 2.3 million 100-ton blocks of solid stone, built for reasons unknown, has struck and eluded all who have taken the time to wonder the purpose of its existence, coming to represent impenetrable yet wondrous mystery incarnate.<sup>3</sup>

<sup>3</sup> As to the incorporation of pi into the design see Tompkins (1971:70) “Taylor then discovered that if he divided the perimeter of the Pyramid by twice its height, it gave him a quotient of 3.144, remarkably close to the value of pi, which is computed as 3.14159+. In other words, the height of the Pyramid appeared to be in relation to the perimeter of its base as the radius of a circle is to its circumference.” In accord see DeSalvo (2008:72-73), Skinner (2006:116-119), Dunn, (1998:59).

As to phi see Tompkins (1971:190) “(T)he Pyramid was designed to incorporate not only the pi proportion by another and even more useful constant proportion, known in the Renaissance as the Golden Section, designated in modern times by the Greek letter  $\phi$  (pronounced phi) or 1.618. (If the 356 cubits of the Pyramid’s apothem are divided by half the base of 220 cubits, the result is 89/55, or 1.618.)” In accord see Skinner (2006: 119-121), Hemenway (2005:68).

It has been shown that the central quantitative fixture of the economy of the United States is the proportion  $1:\phi$ , as fully revealed in the following diagram and its explicating essay. (Albers & Albers, 2013) In fine, over the course of 14 years the real GNP of the United States increases on average in a  $1 : 1.6180$  ratio. The biologic, mystical, natural, mathematic, etc. associations, benign and otherwise, brought forward by this unexpected yet quite quantifiable fact are yet to be explored fully.

**DIAGRAM 4-7. THE "GNP SPIRAL"**

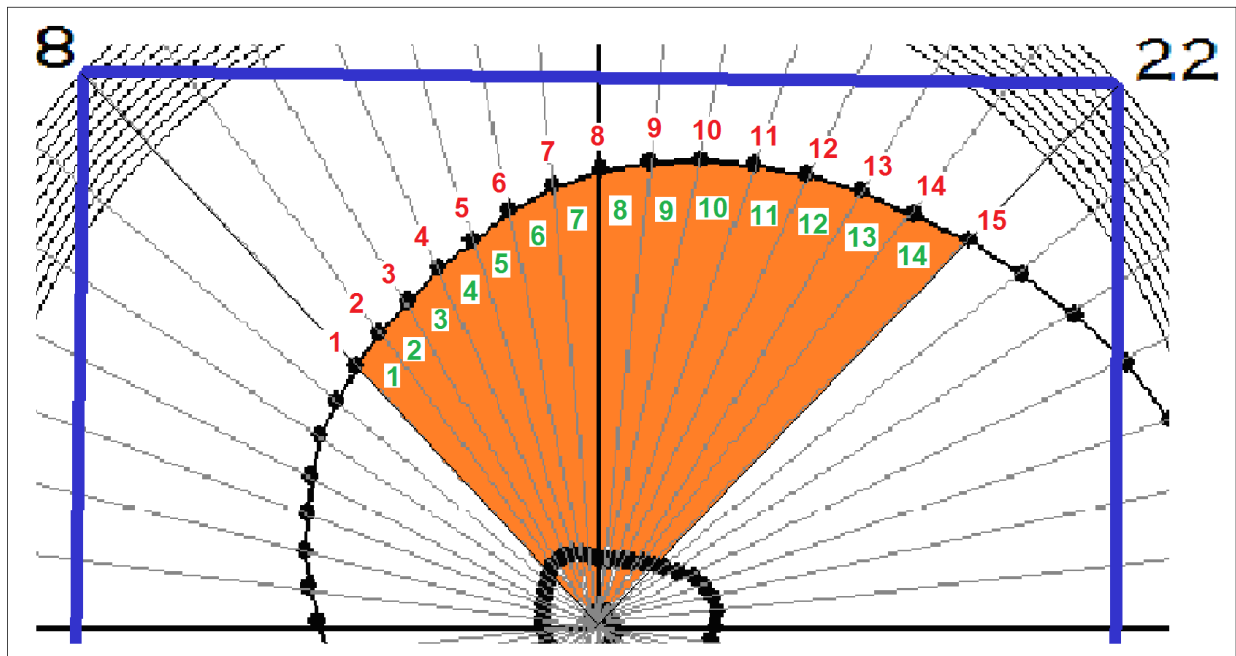


The above spiral, which mimics the spiral of galaxies and shellfish alike, brings forward numerous questions as to the nature of time in social systems. Here let us note that one of these aspects is that the running of a period of time, like the running of a race, suggests both masculine and feminine numbers.

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In some apparent opposition to both see Livio, (2002:53-61). Calculating a difference from phi at “less than 0.1%” (at 56) and “differing (from pi) only by about 0.05%” (at 58), he argues that these proportions are not those of an original design by the builder of the Great Pyramid. “(I)t is highly unlikely that either the ancient Babylonians or the ancient Egyptians discovered the Golden Ratio and its properties; this task was left for the Greek mathematicians.” (at 61)





In so far as the race begins with a starting line and ends with a finish line, then the number of lines counted will be one more than the spaces held between the lines. In the above case highlighted in orange we count 15 lines creating 14 spaces. The fourteen spaces themselves contain a specific number days. To begin the count of days we start at the first day, indicating the starting line of the race. It is, however, the second line, not the first, which represents the end of the first year.

Consequently the period of time might be measured in feminine numbers as 14/15 (counting the time held within the boundaries).

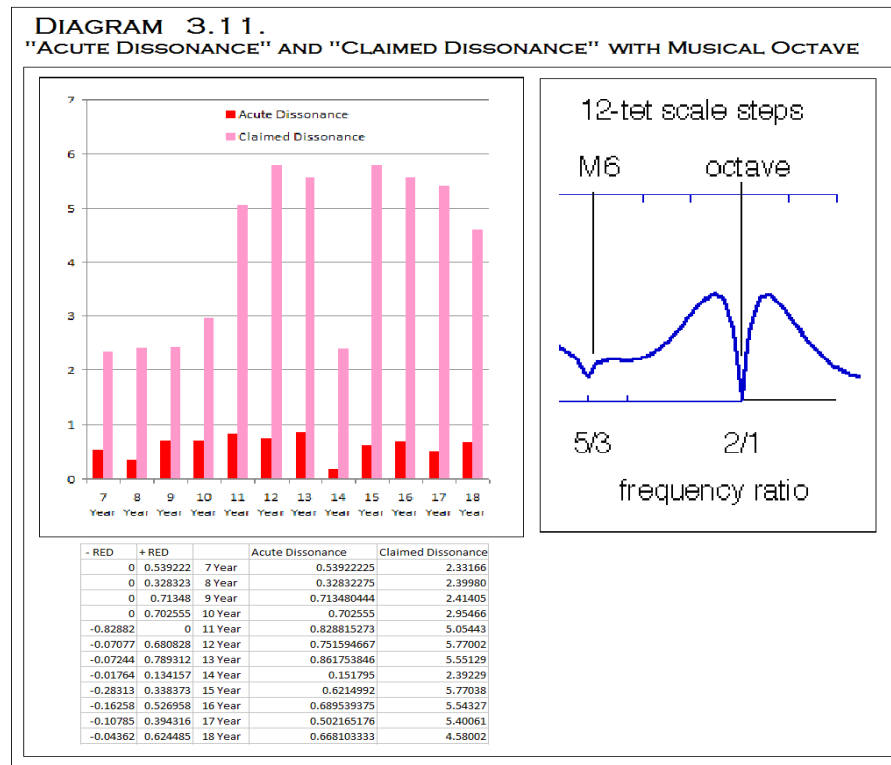
Conversely we may consider the same period as stated in masculine numbers at its inverse, 15/14 (counting the number of boundaries holding the time period).

The two sets of numbers will equal “one” if multiplied together in a proper inverse. But if a Jane Austen multiplicative inverse is intended, the result may be considerably different.

The spiral itself is based upon an harmonic interpretation of the Kondratiev Wave, an historic “Long Wave” of political-economy of precisely 56 years in duration, an interpretation which hails from no less of a figure in mathematics than Pythagoras himself. Through the distinct similarity which ratios of U.S. real GNP using various “spreads of years” have with octaves of musical harmony, one may determine “octaves” of mathematic association within the economic data itself, falling at spreads of 14 years. This is consonant with the onset of reproductive capabilities within the American citizenry; moreover it presents associations of both economics as well as politics.



Using the above model – “the GNP Spiral” – repetitions of constitutional amendment in the lower left quadrant stand at a 18 liberal : 3 conservative ratio in relation to the upper right quadrant. Moreover the Golden Mean and its association with  $\phi = 1.6180...$  is stated to within 3.4 parts of 10,000 – and under even more exacting analysis at 5.3 parts of 100,000 – with an explained steady-state rate of growth between 3.496 and 3.499 percent annually.



If an association exists between the economy of the United States and the Great Pyramid through their mutual apparent interest in the Golden Mean and its  $1 : \phi$  ratio, then one must inquire whether there might be also a further fixed association with the ratio  $1 : \pi$ , a mathematical relationship which must occur if truth be as interesting as fiction.

And so it is with mingled suspicion and suspense that one reads that in 1962 the august Arthur Okun discovered “Okun’s Law,” a clear, fixed and unexplained 3:1 relationship between a percentage increase in GNP production and percentage increase in the rate of employment. Might this 3:1 proportion be, in reality, a  $\pi:1$  proportion?, thereby rendering unto the economists and the economy of the United States the infinite heaven-ward of impossibility?

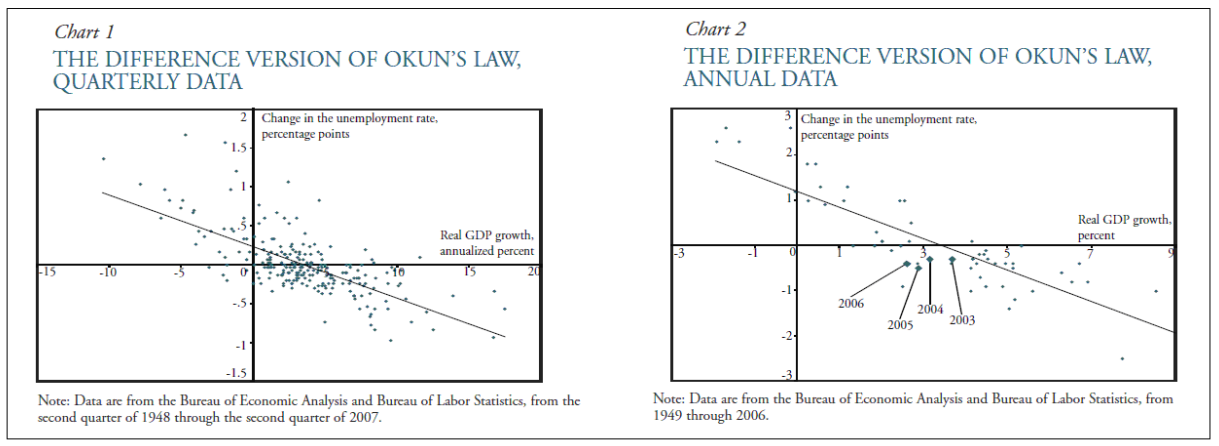
## An Officer and a Gentleman

Stated specifically, “Okun’s Law” notes that for every three percentage points of increase in real GNP the rate of employment increases by one percentage point, and that decreases of both take place at the same rate.<sup>1</sup> This 3 : 1 proportion is generally referred to using a double negative, i.e. an increase of three percent in real GNP will lead to a one percent *decrease* in the rate of *unemployment*. Although first stated by Arthur Okun, at the time senior economist of President Kennedy’s Council of Economic Advisors, Okun’s Law has taken on a legend of its own, being termed “one of the most reliable empirical regularities in macroeconomics.”<sup>1</sup> (Tobin, 1983)

Dr. Edward Knotek’s article “How Useful Is Okun’s Law?” (Kansas City Federal Reserve *Economic Review*, 2007, in the public domain) proposes that Okun’s Law is, at best, a helpful rule of thumb. As the title of the article suggests directly, Dr. Knotek describes in detail our present understanding of Okun’s Law as both a mathematic equation and as a policy tool.

To make the point of his article Dr. Knotek organizes data sets which follow mainstream econometric methods as applied to well-known and easily available federal data bases covering a 60 year period of American economic history, i.e. the second quarter of 1947 through the third quarter of 2007. Charts One and Two graph the quarterly and annual data sets supporting the regularity of the relationship between changes in the size of real GNP (x-axis) and the corresponding effect this has on the rate of employment (y-axis).

**DIAGRAM 2-2.**  
**CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"**



Knotek's Quarterly Data, Email of November 30, 2011			Knotek's Annual Data, Email of July 28, 2011		
Quarterly Figure					
dy	du_avg		dy	du	
1948.2	7.242819	-0.06667	1949	-1.69481	2.6
1948.3	2.33069	0.1	1950	13.43641	-2.3
1948.4	0.946452	0.066667	1951	5.165233	-1.2
1949.1	-5.8502	0.833333	1952	5.112117	-0.4
1949.2	-1.17043	1.2	1953	0.425678	1.8
1949.3	4.571555	0.833333	1954	2.694275	0.5
1949.4	-0.41886	0.266667	1955	6.523389	-0.8
1950.1	17.45697	-0.56667	1956	1.834944	0
1950.2	12.44685	-0.83333	1957	0.26241	1
1950.3	16.62782	-0.93333	1958	2.420938	1
1950.4	7.493361	-0.4	1959	4.88075	-0.9
1951.1	4.934907	-0.73333	1960	0.552262	1.3
1951.2	6.972653	-0.4	1961	6.283822	-0.6
1951.3	8.229388	0.066667	1962	4.111255	-0.5
1951.4	0.681763	0.2	1963	5.321168	0
1952.1	4.241951	-0.3	1964	5.12163	-0.5
1952.2	0.264934	-0.1	1965	8.51134	-0.1
1952.3	2.629817	0.266667	1966	4.310711	-0.2
1952.4	13.80097	-0.4	1967	2.460902	0
1953.1	7.75058	-0.13333	1968	4.940026	0.4
1953.2	3.069825	-0.13333	1969	2.012459	0.1
1953.3	-2.39934	0.166667	1970	-0.17258	2.6
1953.4	-6.16277	0.966667	1971	4.470982	-0.1
1954.1	-1.95385	1.566667	1972	6.894269	-0.8
1954.2	0.372631	0.533333	1973	4.15843	-0.3
1954.3	4.495923	0.166667	1974	-1.92989	2.3
1954.4	8.153291	-0.63333	1975	2.539113	1
1955.1	12.02633	-0.6	1976	4.247578	-0.4
1955.2	6.710869	-0.33333	1977	5.032064	-1.4
1955.3	5.443469	-0.3	1978	6.689095	-0.4
1955.4	2.148374	0.133333	1979	1.310001	0
1956.1	-1.85749	-0.2	1980	-0.04995	1.2
1956.2	3.205905	0.166667	1981	1.17833	1.3
1956.3	-0.47861	-0.06667	1982	-1.39834	2.3
1956.4	6.686428	-1E-15	1983	7.720914	-2.5
1957.1	2.436114	-0.2	1984	5.579107	-1
1957.2	-0.9875	0.166667	1985	4.171185	-0.3
1957.3	3.962372	0.133333	1986	2.842924	-0.4
1957.4	-4.16299	0.7	1987	4.481766	-0.9
1958.1	-10.4351	1.366667	1988	3.659866	-0.4
1958.2	2.388603	0.066667	1989	2.661858	0.1
1958.3	9.560821	-0.03333	1990	0.654255	0.9
1958.4	9.624235	-0.96667	1991	1.090873	1
1959.1	7.871279	-0.53333	1992	4.145874	0.1
1959.2	10.93633	-0.73333	1993	2.505805	-0.9
1959.3	-0.30911	0.166667	1994	4.113965	-1
1959.4	1.42571	0.333333	1995	2.017204	0.1
1960.1	9.202709	-0.46667	1996	4.420611	-0.2
1960.2	-1.98708	0.1	1997	4.342077	-0.7
1960.3	0.62426	0.3	1998	4.510997	-0.3
1960.4	-5.08214	0.733333	1999	4.698444	-0.4
1961.1	2.445174	0.533333	2000	2.239662	-0.1
1961.2	7.728866	0.2	2001	0.225533	1.8
1961.3	6.636591	-0.23333	2002	1.847874	0.3
1961.4	8.42734	-0.56667	2003	3.676776	-0.3
1962.1	7.385135	-0.56667	2004	3.40021	-0.3
1962.2	4.446892	-0.1	2005	3.14980	

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This first table states the size of GDP as measured quarterly. These numbers form the basis for calculating GDP growth. Annual GDP growth is calculated as  $100 * ((\text{GDP in the fourth quarter of this year}) / (\text{GDP in the fourth quarter of last year}) - 1)$ . Quarterly GDP figures are annualized according to the formula provided by the Bureau of Economic Analysis.<sup>4</sup>

Table 1.1.6. Real Gross Domestic Product, Chained Dollars (Billions of chained (2000) dollars); Seasonally adjusted at annual rates Quarterly data from 1947 to 2007 Bureau of Economic Analysis Data published September 27, 2007 File created 9/26/2007 9:47:04 AM							Table 1.1.6. Real Gross Domestic Product, Chained Dollars (Billions of chained (2000) dollars); Seasonally adjusted at annual rates Quarterly data from 1947 to 2007 Bureau of Economic Analysis Data published September 27, 2007 File created 9/26/2007 9:47:04 AM							Table 1.1.6. Real Gross Domestic Product, Chained Dollars (Billions of chained (2000) dollars); Seasonally adjusted at annual rates Quarterly data from 1947 to 2007 Bureau of Economic Analysis Data published September 27, 2007 File created 9/26/2007 9:47:04 AM							Table 1.1.6. Real Gross Domestic Product, Chained Dollars (Billions of chained (2000) dollars); Seasonally adjusted at annual rates Quarterly data from 1947 to 2007 Bureau of Economic Analysis Data published September 27, 2007 File created 9/26/2007 9:47:04 AM													
Year	Quarter	GDP	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate (Current Quarter divided by Previous Quarter) <sup>14</sup> minus 1	Copy of Previous Column (Multiply by 100)	F a Percent (Multiply by 100)	Year	Quarter	GDP	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate (Current Quarter divided by Previous Quarter) <sup>14</sup> minus 1	Copy of Previous Column (Multiply by 100)	F a Percent (Multiply by 100)	Year	Quarter	GDP	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate (Current Quarter divided by Previous Quarter) <sup>14</sup> minus 1	Copy of Previous Column (Multiply by 100)	F a Percent (Multiply by 100)	Year	Quarter	GDP	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate (Current Quarter divided by Previous Quarter) <sup>14</sup> minus 1	Copy of Previous Column (Multiply by 100)	F a Percent (Multiply by 100)							
1947	1	1,570.5					1963	1	2,775.9	1.013	0.01345	0.01345	1.3448	1978	1	4,838.8	1.001	0.01294	0.01294	1.2939	1994	1	7,715.1	1.010	0.04132	0.04132	4.1319	2007	1	14,152.0	1.009	0.02250	0.02250	2.2500
1947	2	1,568.7	0.999	-0.00058	-0.00058	-0.05767	1963	2	2,816.6	1.013	0.01345	0.01345	1.3448	1978	2	5,021.2	1.039	0.16722	0.16722	16.7220	1994	2	7,815.7	1.013	0.05319	0.05319	5.3195	2007	2	14,152.0	1.009	0.02250	0.02250	2.2500
1947	3	1,568.9	1.000	0.00018	0.00018	0.01137	1963	3	2,815.5	1.011	0.01144	0.01144	1.1447	1978	3	5,016.7	1.010	0.00902	0.00902	0.9009	1994	3	7,815.7	1.010	0.02250	0.02250	2.2500	2007	3	14,152.0	1.009	0.02250	0.02250	2.2500
1947	4	1,568.9	1.000	0.00018	0.00018	0.01137	1963	4	2,815.5	1.011	0.01144	0.01144	1.1447	1978	4	5,016.7	1.010	0.00902	0.00902	0.9009	1994	4	7,815.7	1.010	0.02250	0.02250	2.2500	2007	4	14,152.0	1.009	0.02250	0.02250	2.2500
1948	1	1,644.6	1.054	0.05423	0.05423	5.4238	1964	1	2,904.8	1.022	0.02074	0.02074	2.0743	1979	1	5,147.4	1.002	0.00891	0.00891	0.8908	1995	1	7,973.7	1.003	0.01116	0.01116	1.1163	2008	1	14,152.0	1.009	0.02250	0.02250	2.2500
1948	2	1,654.1	1.006	0.00231	0.00231	0.2309	1964	2	2,904.8	1.022	0.02074	0.02074	2.0743	1979	2	5,147.4	1.001	0.00891	0.00891	0.8908	1995	2	7,973.7	1.003	0.01116	0.01116	1.1163	2008	2	14,152.0	1.009	0.02250	0.02250	2.2500
1948	3	1,658.9	1.002	0.00094	0.00094	0.05445	1964	3	2,904.8	1.022	0.02074	0.02074	2.0743	1979	3	5,147.4	1.001	0.00891	0.00891	0.8908	1995	3	7,973.7	1.003	0.01116	0.01116	1.1163	2008	3	14,152.0	1.009	0.02250	0.02250	2.2500
1948	4	1,658.9	1.002	0.00094	0.00094	0.05445	1964	4	2,904.8	1.022	0.02074	0.02074	2.0743	1979	4	5,147.4	1.001	0.00891	0.00891	0.8908	1995	4	7,973.7	1.003	0.01116	0.01116	1.1163	2008	4	14,152.0	1.009	0.02250	0.02250	2.2500
1949	1	1,813.2	1.105	0.10550	0.10550	10.5500	1965	1	3,025.5	1.024	0.02154	0.02154	2.1540	1980	1	5,307.4	1.001	0.00891	0.00891	0.8908	1996	1	8,053.1	1.008	0.02300	0.02300	2.3000	2009	1	14,152.0	1.009	0.02250	0.02250	2.2500
1949	2	1,828.4	1.010	0.00910	0.00910	0.51144	1965	2	3,025.5	1.024	0.02154	0.02154	2.1540	1980	2	5,307.4	1.001	0.00891	0.00891	0.8908	1996	2	8,053.1	1.008	0.02300	0.02300	2.3000	2009	2	14,152.0	1.009	0.02250	0.02250	2.2500
1949	3	1,828.4	1.010	0.00910	0.00910	0.51144	1965	3	3,025.5	1.024	0.02154	0.02154	2.1540	1980	3	5,307.4	1.001	0.00891	0.00891	0.8908	1996	3	8,053.1	1.008	0.02300	0.02300	2.3000	2009	3	14,152.0	1.009	0.02250	0.02250	2.2500
1949	4	1,828.4	1.010	0.00910	0.00910	0.51144	1965	4	3,025.5	1.024	0.02154	0.02154	2.1540	1980	4	5,307.4	1.001	0.00891	0.00891	0.8908	1996	4	8,053.1	1.008	0.02300	0.02300	2.3000	2009	4	14,152.0	1.009	0.02250	0.02250	2.2500
1950	1	1,915.8	1.053	0.05328	0.05328	5.3282	1966	1	3,172.3	1.024	0.02154	0.02154	2.1540	1981	1	5,307.4	1.001	0.00891	0.00891	0.8908	1997	1	8,053.1	1.008	0.02300	0.02300	2.3000	2010	1	14,152.0	1.009	0.02250	0.02250	2.2500
1950	2	1,915.8	1.053	0.05328	0.05328	5.3282	1966	2	3,172.3	1.024	0.02154	0.02154	2.1540	1981	2	5,307.4	1.001	0.00891	0.00891	0.8908	1997	2	8,053.1	1.008	0.02300	0.02300	2.3000	2010	2	14,152.0	1.009	0.02250	0.02250	2.2500
1950	3	1,915.8	1.053	0.05328	0.05328	5.3282	1966	3	3,172.3	1.024	0.02154	0.02154	2.1540	1981	3	5,307.4	1.001	0.00891	0.00891	0.8908	1997	3	8,053.1	1.008	0.02300	0.02300	2.3000	2010	3	14,152.0	1.009	0.02250	0.02250	2.2500
1950	4	1,915.8	1.053	0.05328	0.05328	5.3282	1966	4	3,172.3	1.024	0.02154	0.02154	2.1540	1981	4	5,307.4	1.001	0.00891	0.00891	0.8908	1997	4	8,053.1	1.008	0.02300	0.02300	2.3000	2010	4	14,152.0	1.009	0.02250	0.02250	2.2500
1951	1	1,981.1	1.037	0.03723	0.03723	3.7235	1967	1	3,444.3	1.000	0.00023	0.00023	0.02310	1982	1	5,117.1	1.001	0.00891	0.00891	0.8908	1998	1	8,053.1	1.008	0.02300	0.02300	2.3000	2011	1	14,152.0	1.009	0.02250	0.02250	2.2500
1951	2	1,981.1	1.037	0.03723	0.03723	3.7235	1967	2	3,444.3	1.000	0.00023	0.00023	0.02310	1982	2	5,117.1	1.001	0.00891	0.00891	0.8908	1998	2	8,053.1	1.008	0.02300	0.02300	2.3000	2011	2	14,152.0	1.009	0.02250	0.02250	2.2500
1951	3	1,981.1	1.037	0.03723	0.03723	3.7235	1967	3	3,444.3	1.000	0.00023	0.00023	0.02310	1982	3	5,117.1	1.001	0.00891	0.00891	0.8908	1998	3	8,053.1	1.008	0.02300	0.02300	2.3000	2011	3	14,152.0	1.009	0.02250	0.02250	2.2500
1951	4	1,981.1	1.037	0.03723	0.03723	3.7235	1967	4	3,444.3	1.000	0.00023	0.00023	0.02310	1982	4	5,117.1	1.001	0.00891	0.00891	0.8908	1998	4	8,053.1	1.008	0.02300	0.02300	2.3000	2011	4	14,152.0	1.009	0.02250	0.02250	2.2500
1952	1	2,064.7	1.039	0.03942	0.03942	3.9429	1968	1	3,596.7	1.001	0.00091	0.00091	0.09116	1983	1	5,253.8	1.001	0.00925	0.00925	0.92545	1999	1	8,315.5	1.008	0.02340	0.02340	2.3404	2012	1	14,152.0	1.009	0.02250	0.02250	2.2500
1952	2	2,064.7	1.039	0.03942	0.03942	3.9429	1968	2	3,596.7	1.001	0.00091	0.00091	0.09116	1983	2	5,253.8	1.001	0.00925	0.00925	0.92545	1999	2	8,315.5	1.008	0.02340	0.02340	2.3404	2012	2	14,152.0	1.009	0.02250	0.02250	2.2500
1952	3	2,064.7	1.039	0.03942	0.03942	3.9429	1968	3	3,596.7	1.001	0.00091	0.00091	0.09116	1983	3	5,253.8	1.001	0.00925	0.00925	0.92545	1999	3	8,315.5	1.008	0.02340	0.02340	2.3404	2012	3	14,152.0	1.009	0.02250	0.02250	2.2500
1952	4	2,064.7	1.039	0.03942	0.03942	3.9429	1968	4	3,596.7	1.001	0.00091	0.00091	0.09116	1983	4	5,253.8	1.001	0.00925	0.00925	0.92545	1999	4	8,315.5	1.008	0.02340	0.02340	2.3404	2012	4	14,152.0	1.009	0.02250	0.02250	2.2500
1953	1	2,185.4	1.059	0.05929	0.05929	5.9294	1969	1	3,784.2	1.001	0.00091	0.00091	0.09116	1984	1	5,358.5	1.001	0.00940	0.00940	0.94040	2000	1	8,567.1	1.008	0.02380	0.02380	2.3804	2013	1	14,152.0	1.009	0.02250	0.02250	2.2500
1953	2	2,185.4	1.059	0.05929	0.05929	5.9294	1969	2	3,784.2	1.001	0.00091	0.00091	0.09116	1984	2	5,358.5	1.001	0.00940	0.00940	0.94040	2000	2	8,567.1	1.008	0.02380	0.02380	2.3804	2013	2	14,152.0	1.009	0.02250	0.02250	2.2500
1953	3	2,185.4	1.059	0.05929	0.05929	5.9294	1969	3	3,784.2	1.001	0.00091	0.00091	0.09116	1984	3	5,358.5	1.001	0.00940	0.00940	0.94040	2000	3	8,567.1	1.008	0.02380	0.02380	2.3804	2013	3	14,152.0	1.009	0.02250	0.02250	2.2500
1953	4	2,185.4	1.059	0.05929	0.05929	5.9294	1969	4	3,784.2	1.001	0.00091	0.00091	0.09116	1984	4	5,358.5	1.001	0.00940	0.00940	0.94040	2000	4	8,567.1	1.008	0.02380	0.02380	2.3804	2013	4	14,152.0	1.009	0.02250	0.02250	2.2500
1954	1	2,242.4	1.029	0.02914	0.02914	2.9145	1970	1	3,916.7	1.000	0.00000	0.00000	0.00000	1985	1	5,409.8	1.001	0.00950	0.00950	0.95040	2001	1	8,667.1	1.008	0.02420	0.02420	2.4204	2014	1	14,152.0	1.009	0.02250	0.02250	2.2500
1954	2	2,242.4	1.029	0.02914	0.02914	2.9145	1970																											

than is possible for annual data. This is quite unlike the nature of a measurement of quarterly and annual employment as we shall see.

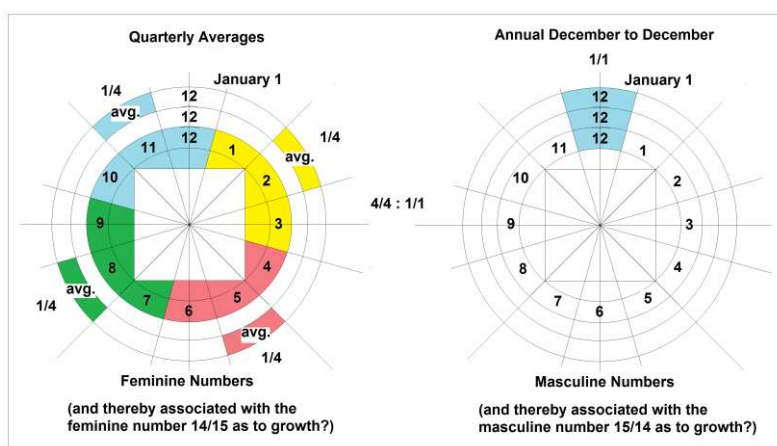
The next table below states the employment rate in months. For annual data, the change in the unemployment rate is the current December minus the previous December. For quarterly data, the change in the unemployment rate is the difference between subsequent quarterly averages.<sup>5</sup>

Monthly Unemployment, Bureau of Labor Statistics													
Labor Force Statistics from the Current Population Survey													

The above chart is of specific interest as it relates to the calculation of quantities of monthly unemployment in both their “feminine” and “masculine” characteristics, or put another way, their “circumferential” and “radial” characteristics.

To make the distinction plain, let us imagine that the march of months within a year was made congruent to the 12 hours on the face of the clock. The manner in which the data for unemployment is collected and analyzed against itself partakes of the circular nature of a unit circle. In this way the average of each quarter is taken and compared with the average of each other quarter. This is portrayed in the left hand side of the following chart.

Contrast this with a single month, chosen from the twelve, and it alone being contrasted with the same month of the following year, and then the following year, and so on.



On the left we have a circumferential relationship between quarterly data which itself relies upon a circular sense of time, a legitimate apportion-izing of something which itself is taken as a “1.” On the right we have a distinctly different and radial view of time, one which does not accept any obvious limitation to its ongoing list of endless Decembers.<sup>6</sup>

Note that the estimation of a “quarterly” rate for unemployment takes as its beginning source of numeric encouragement the idea that it is  $1/4^{\text{th}}$  of something else, specifically a sub-part of a 12-month, four-quarter year. If we were to have a full year specified in quarters then numerically we would be interested in a year stated as  $4/4$  which, according to number theory, would equal a single year.

Conversely the statement of an “annual” rate of unemployment seeks not an association between the data and the year itself, but rather to an *on-going set of years in sequence*. Consequently the rate of one December is compared to the rate of the next December and measured. In contrast to the quarterly data – which by definition is part of some other wholeness – we might state annual data as a repeated sense of “1,” each point repeating itself in endless time, a  $1/1$ .

Here we enter into the intrigues and quiet thoughts of the numbers themselves. Placing both feminine and masculine numbers together we see above a hinted “radius :  $2\pi$ ” relationship

<sup>66</sup> It must be noted, however, that the GNP Spiral assumes a further circular aspect of time applying even to annual data. Consequently the 14/15 association of feminine numbers in this regard, and the 15/14 association with masculine numbers remains a connected aspect of this insight. In short, if the annual data itself falls into a larger circumferential relationship, what relationship might this have to the quarterly data which are, at best, a sub-part of the GNP Spiral and its  $1:\phi$  ratio over a span of 56 years?



between annual and quarterly approaches using a single data set describing unemployment and a second single data set describing GDP growth.

Do the feminine ( $0 < F < 1$ ) numbers maintain a secret relationship with the quarterly employment figures, their circumferential sense of time and the fraction 14/15 as these relate to the GNP Spiral / Kondratiev Wave, perhaps “filling up” the space between moments of time?

Do the masculine ( $1 < M$ ) numbers share an equally hidden relationship with annual employment figures, their radial sense of time and the fraction 15/14 as these relate to the GNP Spiral / Kondratiev Wave, perhaps setting up “boundaries” separating moments of time?

Under what circumstances might these secrets be revealed, secrets which although hidden, tentative and circumspect, might actually bear an inverse relationship of some sort to one another?

The following states the annual measures of GNP as compared with Dr. Knotek.

#### Change in Annual Unemployment vs. Change in Annual GDP

Quarterly GDP, Bureau of Economic Analysis

Table 1.1.6. Real Gross Domestic Product, Chained Dollars

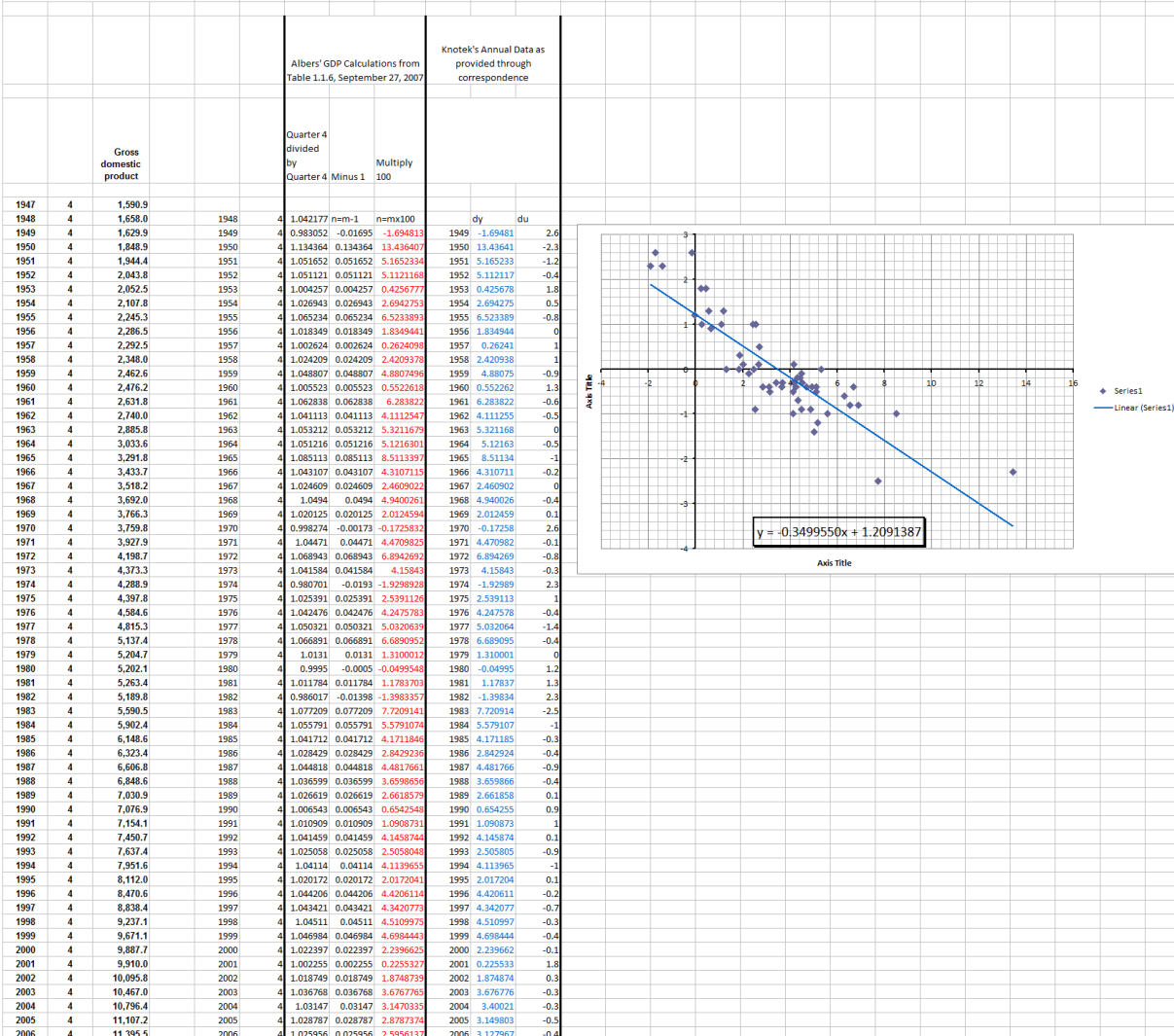
[Billions of chained (2000) dollars]; Seasonally adjusted at annual rates

Quarterly data from 1947 To 2007

Bureau of Economic Analysis

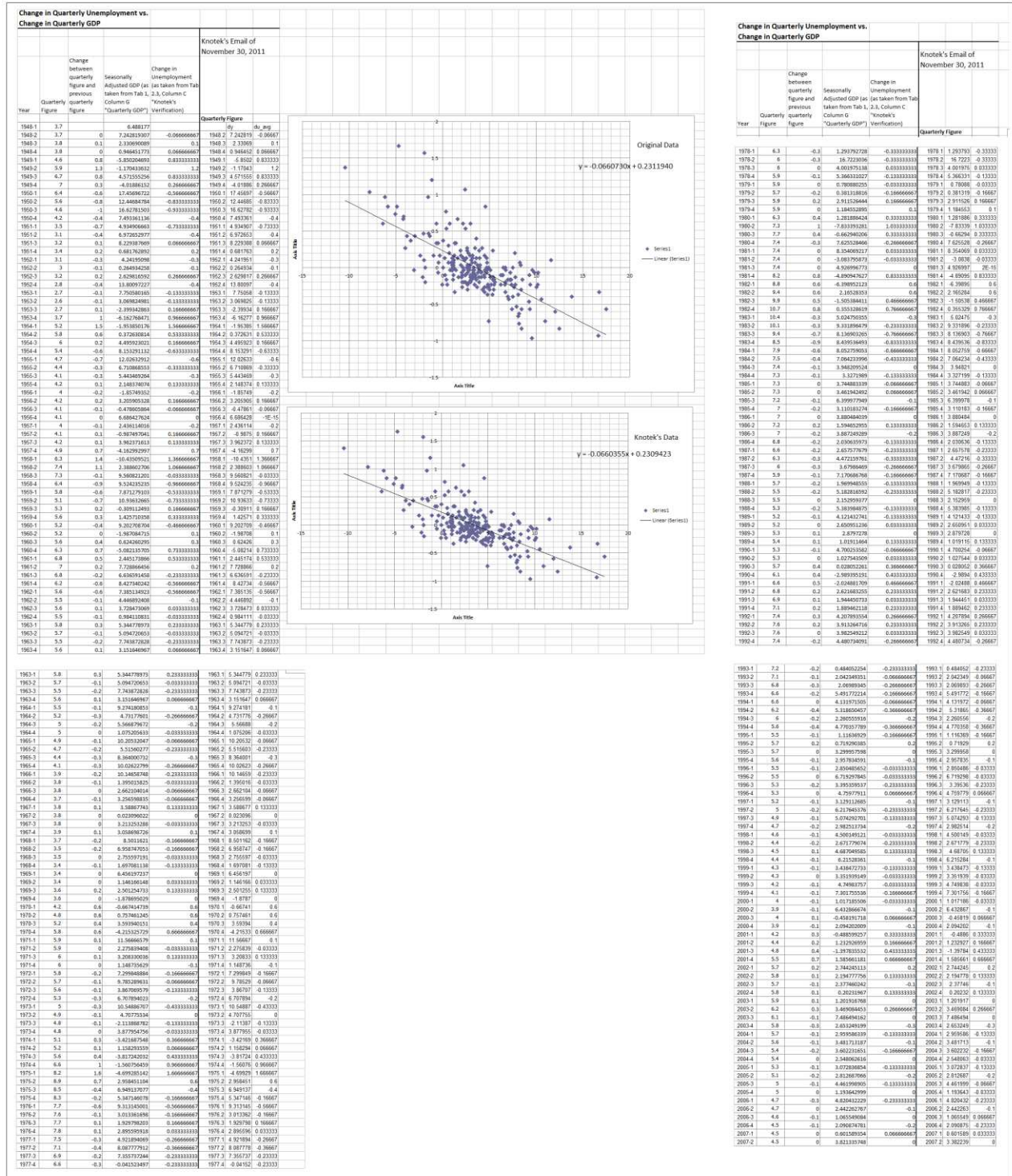
Data published September 27, 2007

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The following states the quarterly data for annualized real GNP and quarterly employment, as contrasted with that of Dr. Knotek.

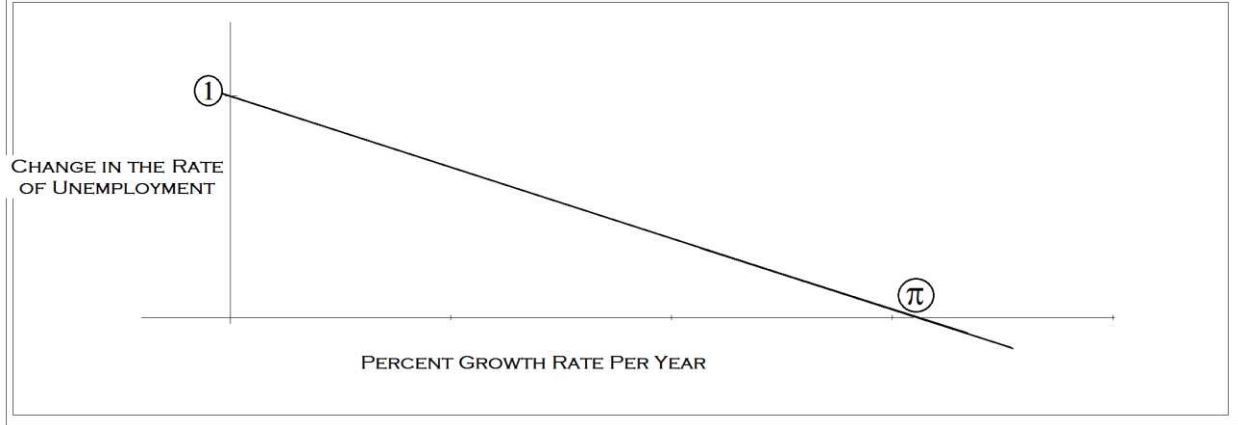


## “The Multiplicative Inverse Surprise”

The relationship which we are anticipating is that a  $1 : \pi$  relationship will exist between a percentage change in the rate of unemployment and the percentage growth of GNP. As the rate of growth increases on the x-axis, the rate of unemployment will go down on the y-axis. Setting this relationship as a straight-forward linear relationship, we have the following.

DIAGRAM 2-13.

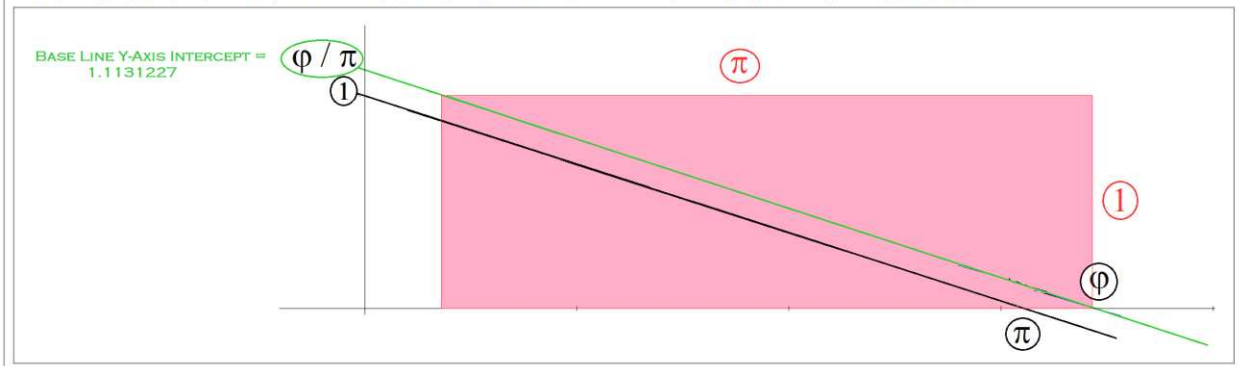
PI : 1 RELATIONSHIP BETWEEN GNP GROWTH AND CHANGE IN THE RATE OF EMPLOYMENT



In order to establish a  $1 : \phi$  proportion over fourteen years the economy of the United States must possess a steady state rate of growth of approximately 3.4969% per year. As one calculates a  $1 : \pi$  exchange between rates of unemployment and GDP growth under Okun's Law, one notices that the slope of the  $1 : \pi$  relationship must remain the same, but that the y-intercept shifts slightly upwards, becoming not “1” but  $3.4969 / \pi = 1.1131227$ .

DIAGRAM 2-14.

PI : 1 RELATIONSHIP BETWEEN GNP GROWTH AND CHANGE IN THE RATE OF EMPLOYMENT USING GOLDEN MEAN RATE OF GROWTH OF 14-YEAR OCTAVES = 3.4969



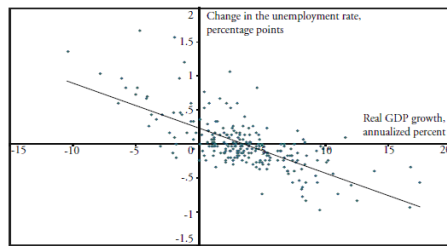
Comparing this to the observed data calculated by Dr. Knotek, one notices that Chart One uses quarterly *growth data* which has been annualized. However quarterly *employment data* is *not* annualized.

DIAGRAM 2-2.

CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"

Chart 1

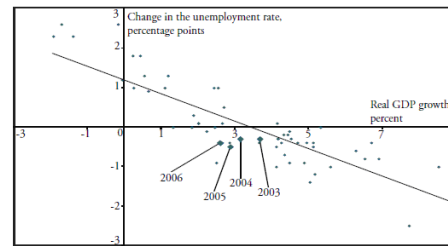
THE DIFFERENCE VERSION OF OKUN'S LAW,  
QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2

THE DIFFERENCE VERSION OF OKUN'S LAW,  
ANNUAL DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

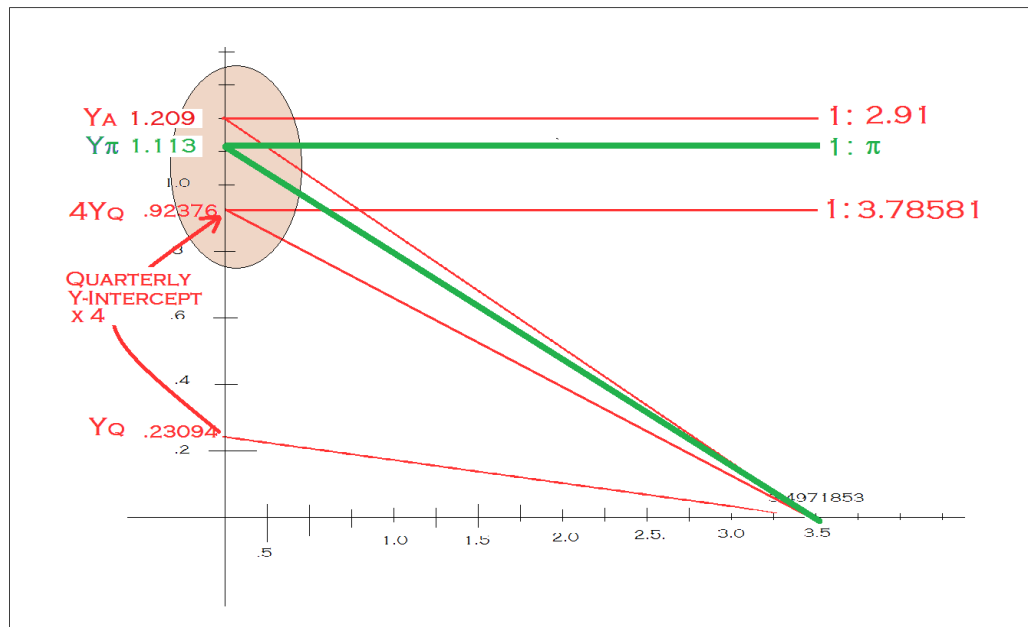
I adjust the trend line for annualized quarterly data by multiplying quarterly employment data by four, thereby “annualizing” quarterly employment data. In this manner annualized quarterly data on growth is matched with “annualized” quarterly data on employment.

DIAGRAM 2-15.

MULTIPLYING QUARTERLY UNEMPLOYMENT RATE X 4

IMPLIED RATIOS BETWEEN  
Y-INTERCEPT AND

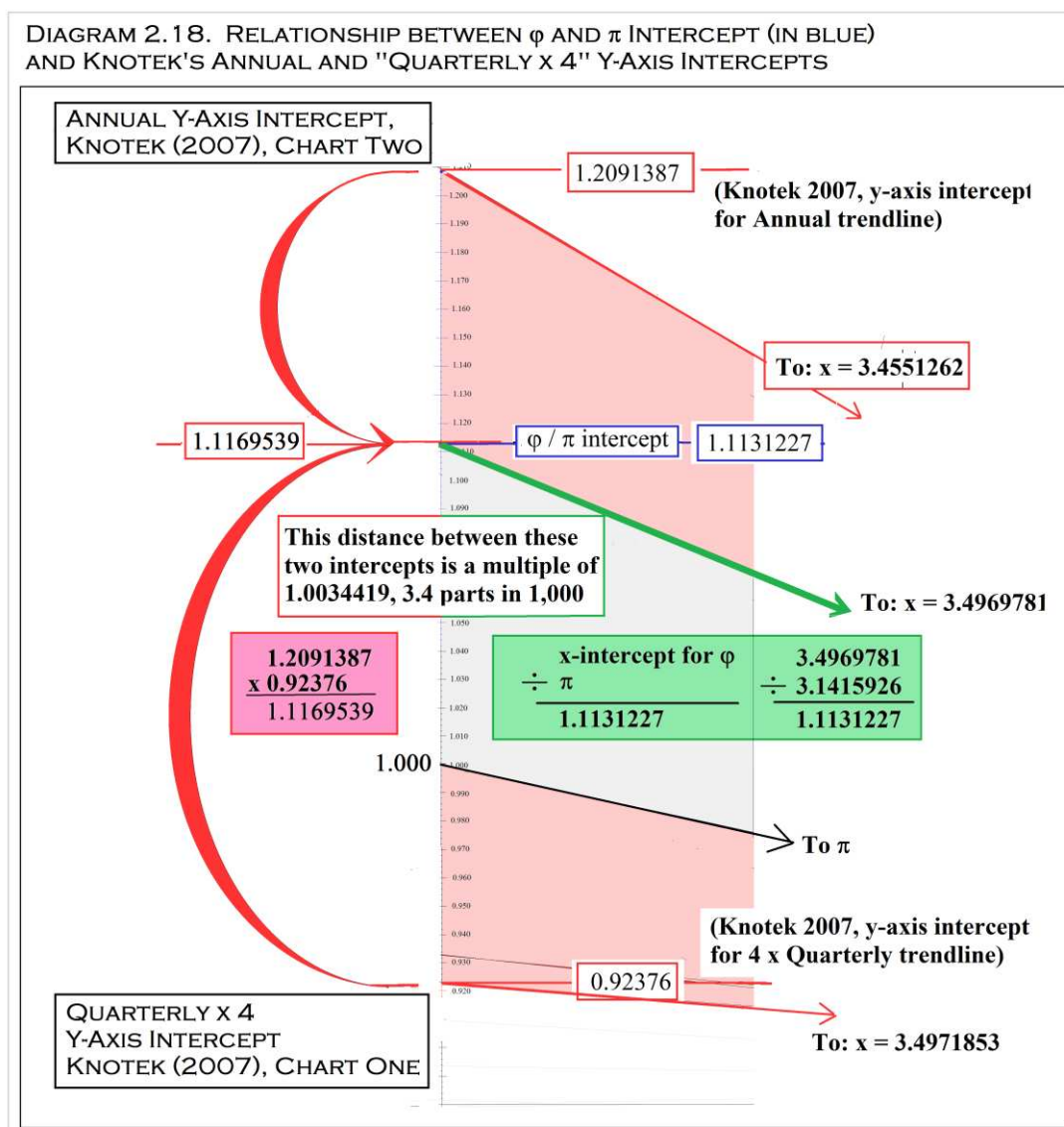
X = 3.4551266 (ANNUAL X-INTERCEPT)  
X = 3.4969781 (GOLDEN MEAN X-INTERCEPT)  
X = 3.4972429 (QUARTERLY X-INTERCEPT)



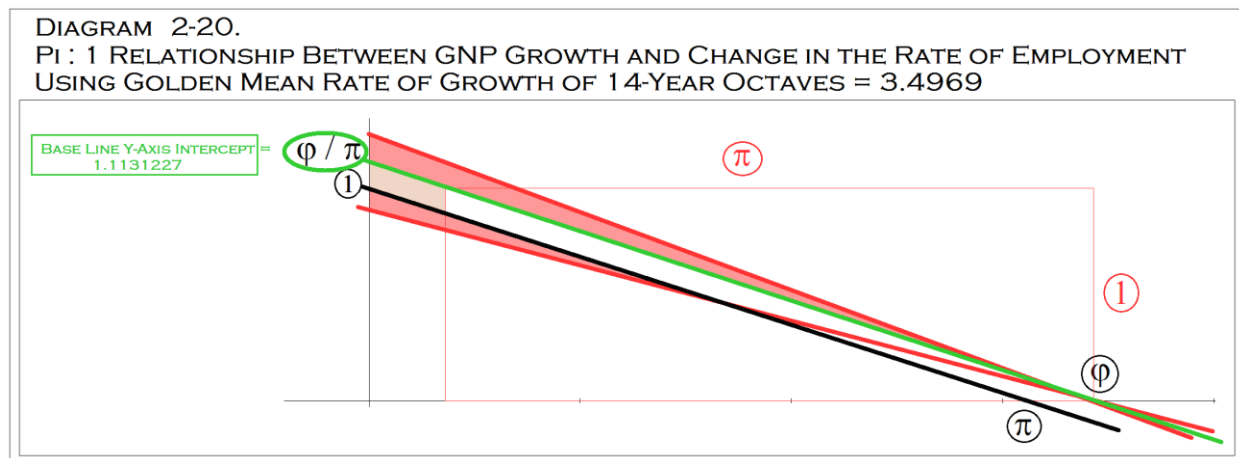
If the steady state rate given for the Golden Mean proportion (3.4969 percent per year) is divided by  $\pi$ , the y-axis intercept is **1.1131227**.

If we accept that the “Annual” y-intercept given in Knotek 2007 as 1.2091387, and that the “4 x Quarterly” y-intercept is 0.92376, we may multiply the two in order to test whether they are inverses around a common point. The multiple of these two intercepts is **1.1169539**. The result is remarkable.

*In short when the growth rate is zero (the y-axis), the y-axis intercepts for “Annual” and “4 x Quarterly” unemployment rate create a “Jane Austen multiplicative inverse” about a progenic y-axis intercept equal to the projected trend line connecting a 1: $\phi$  steady-state rate of growth with a  $\pi$ :1 ratio for Okun’s Law. This calculation is to an accuracy of 0.34%, 3.4 parts in 1,000, or 99.65%. (See chart below)*

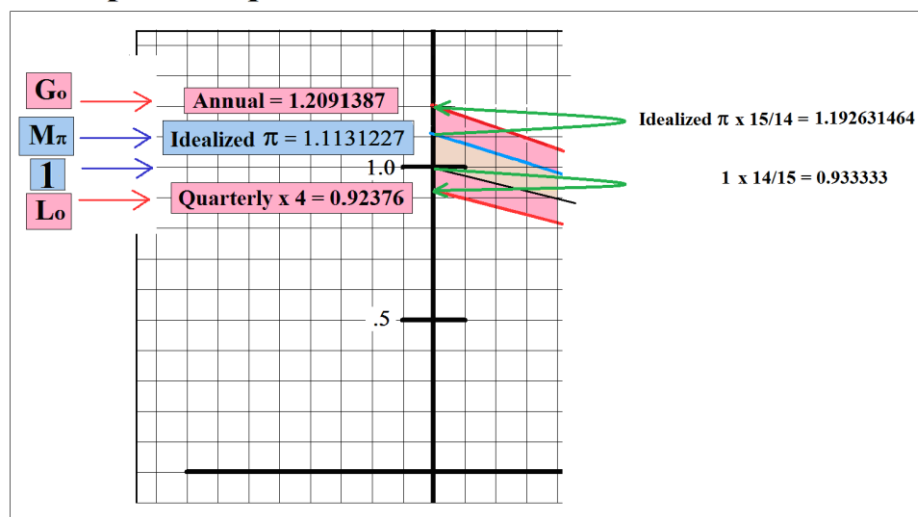


In short, the possibility of two specific sets of numbers – feminine and masculine – as configured in the theory of a Jane Austen multiplicative inverse appears to generate a remarkable understanding of the econometric data underlying Okun’s Law.<sup>7</sup>



The advantages increase considerably if we connect the “1:π / 1:φ” trend line to an analysis of the Kondratiev Wave. We note, as we must, that the progenic  $\pi/\phi$  intercept (“P”) may be constructed from a feminine “14/15 x 1” as combined with a masculine “15/14 x P.” The resulting projections of Annual and Quarterly intercepts lie at variances from Dr. Knotek’s calculations of 1.0% (from the Quarterly unemployment y-axis intercept) and 1.3% (from the Annual unemployment y-axis intercept).

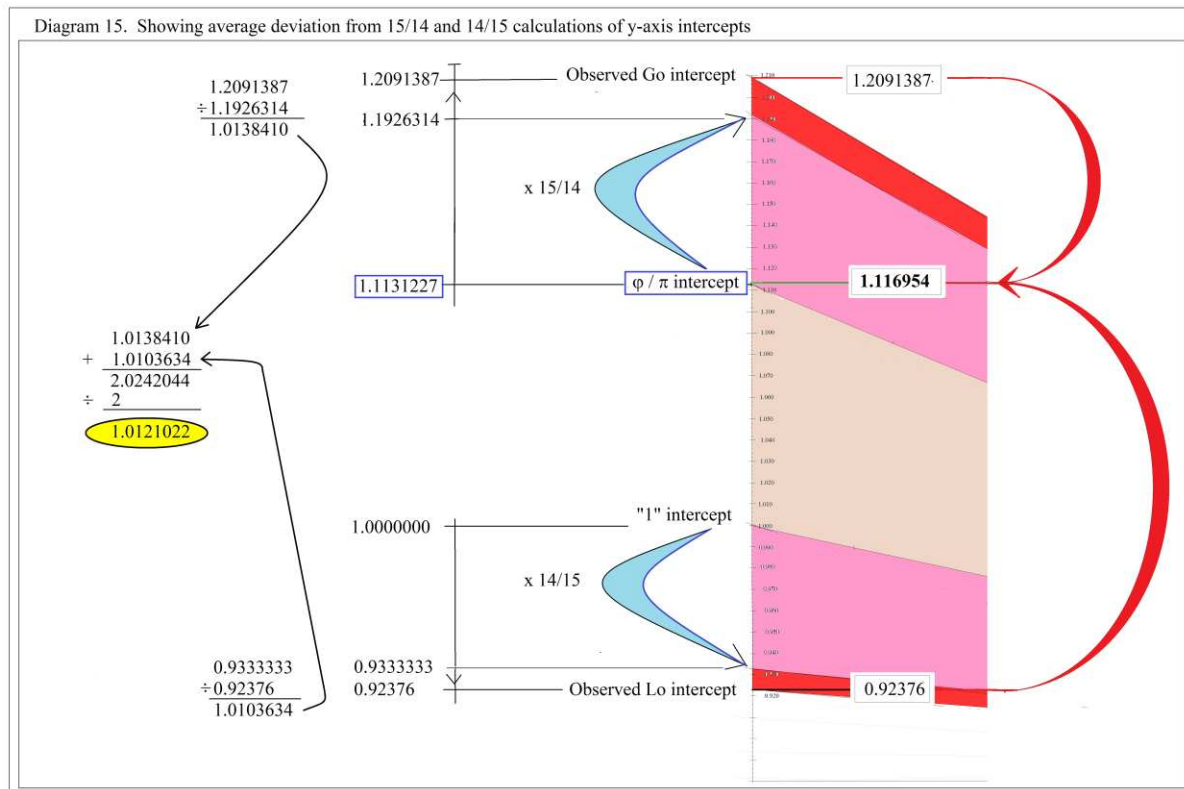
**Diagram 11. Greatest x Least = Middle  
with span of equivalence around  $\pi$  and 1**



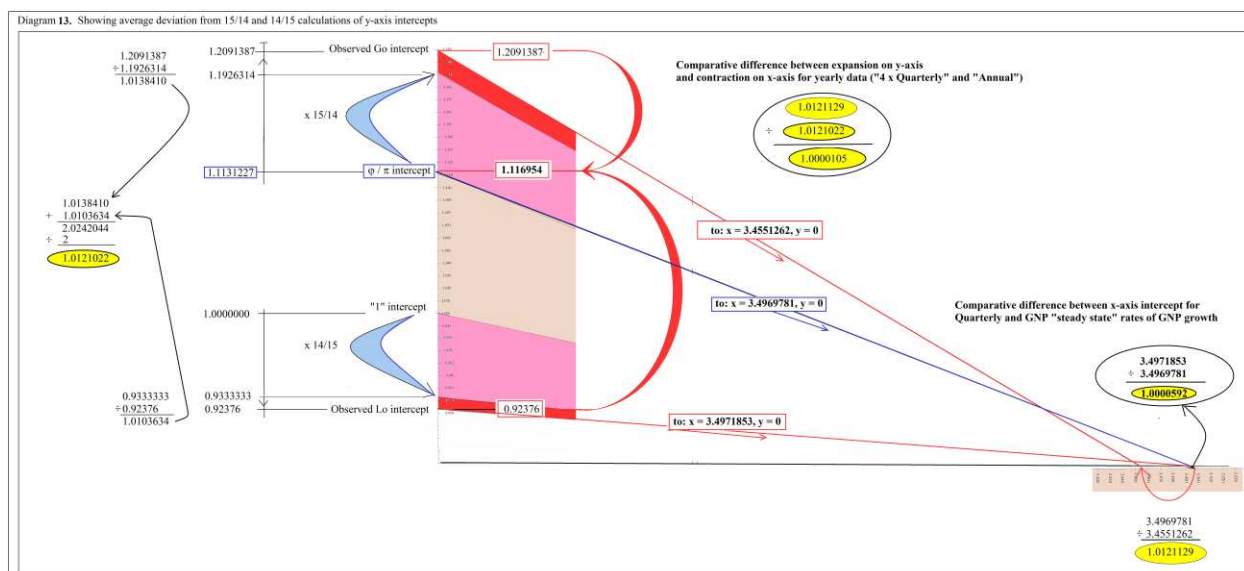
<sup>7</sup> A simple 3:1 ratio, with the same approach used, yields a y-intercept of 1.1656. This is contrasted with a  $\pi/\phi$  intercept of 1.1131 / 1.1656 (at 95.49%) and an observed intercept of 1.1169 / 1.1656 (at 95.81%).



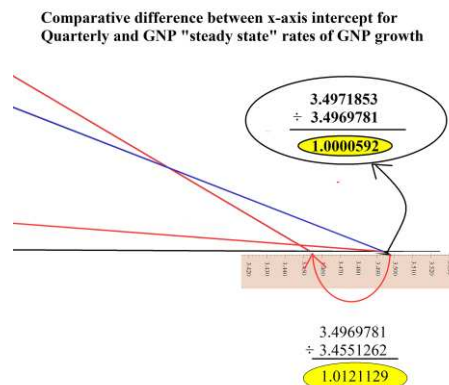
This yields an average expansion of 1.2% beyond the masculine and feminine inverses, or more specifically a multiple of **1.0121022**, in yellow below.



This y-axis deviation balances a similar deviation between growth rates along the x-axis. The steady state rate for Annual Data calculated by Dr. Knotek is 3.4551266. The steady state rate of growth calculated via the GNP Spiral (3.4969781) is greater than this number by a multiple of **1.0121129**, virtually identical to the y-axis deviation stated above.



Let us consider more carefully the three rates we have for a steady state rate of growth, each of which constitutes an x-axis intercept. These are Knotek:Annual (3.4551262), Knotek:Annualized Quarterly (3.4971853) and the GNP Spiral (3.4969781).



Dr. Knotek's data track slightly more than one complete circuit around the 56-year GNP Spiral, i.e. covering the second quarter of 1947 through the third quarter of 2007, a period of 60 years. This data misses the full range of GNP values available from the Department of Commerce (1869 through 1946), a period of 78 years. No unemployment figures are available for this period. Moreover between 1869 and 1947 very large growth rates are found in GNP ratios. These larger ratios are included as a part of the calculation of the GNP Spiral.

Despite the incongruity of data sets Knotek:Annualized Quarterly (3.4971853) is virtually the same as that given for the GNP Spiral (3.4969781).

*When the larger (3.4971853) is divided by the smaller (3.4969781) a multiple of 1.0000592 is found, indicating a proximity between the two numbers of 5.9 parts in 100,000.<sup>8</sup>*

Given the absence of GNP data pre-dating 1947, one might expect that the Knotek:Annual must be smaller than that of the growth rate calculated by the GNP Spiral. Indeed the x-axis intercept for Knotek:Annual (3.4551262) retreats from the expected GNP Spiral x-axis intercept (3.4969781), the second being larger by a multiple of 1.0121129.

As noted previously, this compares to an expansion along the y-axis for unemployment averaging between feminine and masculine components of 1.0121022.

*When the deviation along the x-axis 1.0121129 is divided by the deviation along the y-axis 1.0121022 a multiple of 1.0000105 results. This indicates that a balance between growth and employment along a  $1:\phi / 1:\pi$  trendline is accurate to within 1.05 parts in 100,000. It further suggests that while unemployment states a Jane Austen multiplicative inverse, growth is not figured in such a way.*

<sup>8</sup> This result, as first pointed out by Dr. Knotek in an email of June 24, 2011, was the genesis of the correspondence resulting in this paper.

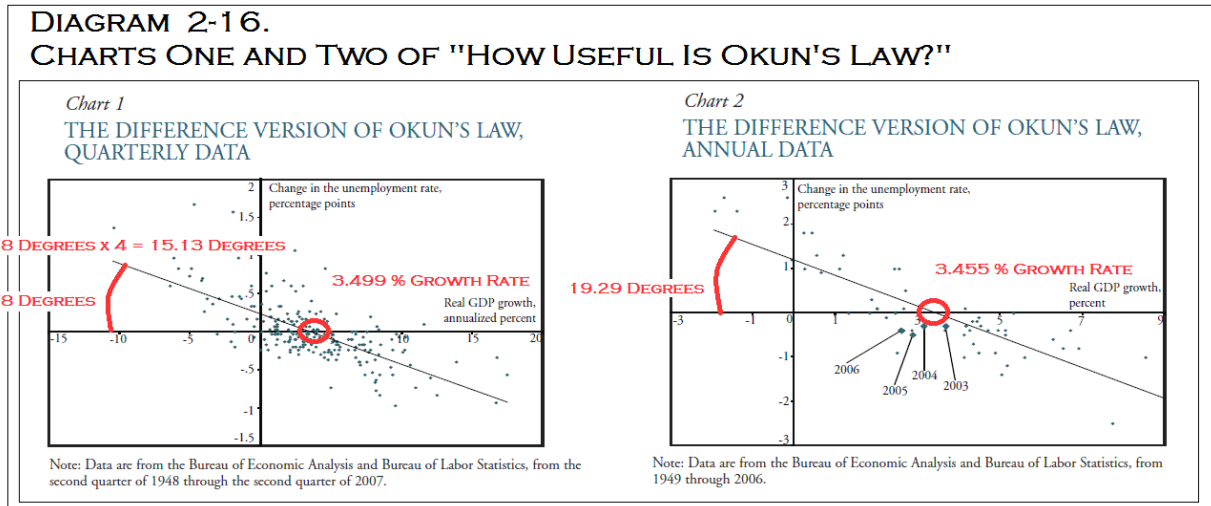


This leads to the following insight as to the operation of the Jane Austen multiplicative inverse and its impact upon the analysis of data surrounding Okun's Law.

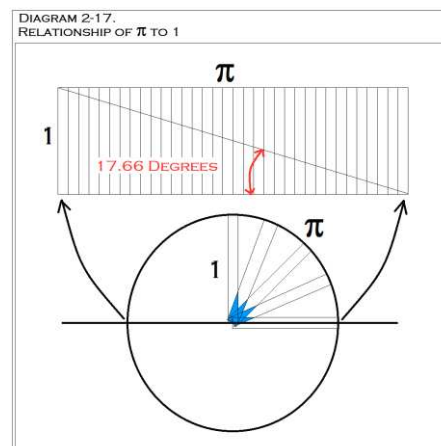
*When change in the rate of unemployment is zero, the rate of growth is seen clearly; there is no inverse at all to found in the growth data.*

*When the growth of GDP is zero, quarterly and annual rates of unemployment at in great flux and we see quite clearly the Jane Austen multiplicative inverse in the unemployment data.*

A second test of the Jane Austen Multiplicative Inverse may be found in the fact that the  $\pi:1$  understanding of Okun's Law generates an angle bisecting that of Charts One and Two to within half a degree. These angles are 15.13 degrees for annualized quarterly data and 19.29 degrees for annual data.



The angle created by the rectangle  $\pi : 1$  bisects these two within one half of one degree, i.e. 17.66 degrees.



In other words, the slope of the angle bisecting the angles given in Charts One and Two is 17.213 degrees, less than half a degree from the slope of 17.66 degrees of a projected relationship between the constant  $\pi$  and 1 as projected by this approach.

## Conclusion

It would appear that Okun's Law is in fact a trigonometrically driven proportion happily demonstrated by a form of number theory engaging the set of feminine ( $0 < F < 1$ ) and masculine ( $1 < M$ ) numbers and using the Jane Austen multiplicative inverse as its foundation stone. This view of the relationships is considerably enhanced the central tenets of the GNP Spiral generate masculine (15/14) and feminine (14/15) fractions which can be used to further interpret the interaction of time upon econometric data, however hidden these relationships might appear.

When further we investigate more closely the intriguing use of unemployment data measured in months to derive both feminine (quarterly, circumferential) and masculine (annual, radial) approaches to precisely the same data set, it is neither surprising nor by chance that the balance thereby derived between unemployment and production lies at 1.05 parts in 100,000 of the hoped for marriage between fact, fancy and econometric data.

Unfortunately at present no theoretic structure has been advanced to explain the apparently long-standing and vital macroeconomic / mathematic relationship given by Okun's Law. The possibility of deriving a theory which explains such an important relationship must be of great interest, i.e. What causal precedents underlie such an important rule of macroeconomics? ... seemingly one of the few macroeconomic observations to ever be denominated a "law" at all.<sup>9</sup>

For without a theory of explanation for the underlying and virtually exact econometric correlations explored herein we are at the mercy of any who charge that numerology rather than logic and science are at play here. The claim of cargo cult science is sure to be made without a theory of causation, a theory satisfactorily explaining to the pygmy why his heathen and mystic incantations will or will not bring the GI, his plane, his emergency landing and his gifts back again.

The answer to this question will be presented in a separate paper.

Scott A. Albers  
February 25, 2013  
Great Falls, Montana

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<sup>9</sup>

See in this regard (Owyang and Sekhposyan, Okun's Law over the Business Cycle, Was the Great Recession All That Different? Federal Reserve Bank of St. Louis Review, September / October, p. 399, 2012) "Many macroeconomic textbooks contain a rule of thumb relating real output growth to changes in the unemployment rate. This relationship, called Okun's law after Okun (1962) typically assigns a 2- to 3- percentage point decrease in real gross domestic product (GDP) growth to a 1-percentage point increase in the unemployment rate. Unlike laws in the physical sciences (e.g. Newton's laws of motion) Okun's law is an (arguably loose) empirical correlation and is, in general, neither theoretically motivated nor strictly adhered to in the data. As many of the reduced-form relationships build strictly on associations and not causation, Okun's law appears to vary depending on the sample period studied."

## Bibliography

Albers, S. and Andrew Albers, (2011). 'The Golden Mean, The Arab Spring and a 10-Step Analysis of American Economic History,' *The Middle East Studies Online Journal*, August 2011, issue 6, volume 3, pp. 199-253.

Albers, S. and Andrew Albers, (2013). 'On the Mathematic Prediction of Crises: Towards a Harmonic Interpretation of the Kondratiev Wave,' *Entelequia*, Spring, 2013.

DeSalvo, J. PhD (2008). *Decoding the Pyramids*, Metro Books, New York.

Dunn, C. (1998). *The Giza Power Plant, Technologies of Ancient Egypt*, Bear and Company Publishing, Santa Fe, New Mexico.

Euclid of Alexandria, Elements.

Goldstein, J. (1988). *Long Cycles: Prosperity and War in the Modern Age*, Yale University Press, New Haven, Conn.

Hemenway, P. (2005). *Divine Proportion, Phi In Art, Nature and Science*, Sterling Publishing Company, New York, NY 10016.

Knotek, E. (2007). 'How Useful Is Okun's Law?' *Economic Review*, Kansas City Federal Reserve, Issue Q IV, pp. 73-103.

Kondratiev, N. D., *The Major Economic Cycles* (in Russian), Moscow, 1925; translated and published as *The Long Wave Cycle* by Richardson & Snyder, New York, 1984.

Korotayev, A. V. and Sergey V. Tsirel, (2010). 'A Spectral Analysis of World GDP Dynamics: Kondratieff Waves, Kuznets Swings, Juglar and Kitchin Cycles in Global Economic Development, and the 2008–2009 Economic Crisis,' *Journal of Structure and Dynamics, Social Dynamics and Complexity*, Institute for Mathematical Behavioral Sciences, University of California at Irvine.

Livio, M. (2002). *The Golden Ratio: The Story of the World's Most Astonishing Number*, Broadway Books, New York.

Owyang, M and T. Sekhposyan, 'Okun's Law over the Business Cycle, Was the Great Recession All That Different?' *Federal Reserve Bank of St. Louis Review*, September / October, p. 399, 2012)

Rucker, R. (1983). *Infinity And The Mind, The Science And Philosophy Of The Infinite*, Bantam Books, December 1983;84-88.

Schumpeter, J. A. (1939). *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*, New York and London: McGraw-Hill Book Company, Inc.

Sethares, W. A. (1992). "Relating Tuning and Timbre," *Experimental Musical Instruments*, September 1992.

Skinner, S. (2006). *Sacred Geometry*, Sterling Publishing, New York, NY. 10016.

Tobin, J. (1983). "Okun, Arthur M." *The New Palgrave Dictionary of Economics*, Vol. 3, pp. 700-701, Macmillan, London.

Tompkins, P. (1971). *Secrets of the Great Pyramid*, Harper and Row, Publishers, New York.

*Historical Statistics of the United States: Colonial Times to 1970, Part I*, United States Department of Commerce, Series F 1-5, "Gross National Product" for the United States between the years 1869-1970 according to 1958 prices.

See also the figures for Real GNP, 1947 to present, maintained by the St. Louis Federal Reserve at <http://research.stlouisfed.org/fred2/series/GNPC96>.